COURSE OBJECTIVES
CHAPTER 1

1 ENGINEERING FUNDAMENTALS

1. Familiarize yourself with engineering plotting, sketching, and graphing techniques so you can use them effectively throughout the remainder of the course.

2. Be able to explain what dependent and independent variables are, notation used, and how relationships are developed between them.

3. Familiarize yourself with the unit systems used in engineering, and know which system is used in Naval Engineering I.

4. Understand unit analysis and be able to use units effectively in doing calculations and in checking your final answer for correctness.

5. Understand significant figures, exact numbers, and the rules for using significant figures in calculations.

6. Obtain a working knowledge of scalars, vectors, and the symbols used in representing them as related to this course.

7. Obtain a working knowledge of forces, moments, and couples.

8. Obtain a working knowledge of the concept of static equilibrium and be able to solve basic problems of static equilibrium.

9. Understand the difference between a distributed force and a resultant force.

10. Know how to determine the first moment of area of a region about its axes.

11. Be able to calculate the geometric centroid of an object.

12. Be able to calculate the second moment of area of a region about an axis.

13. Be able to apply the parallel axis theorem when calculating the second moment of area of a region.

14. Know how to do linear interpolation.

15. Be able to name and describe the six degrees of freedom of a floating ship, and know which directions on a ship are associated with the X, Y, and Z axes.

16.	Know and be able to discuss the following terms as they relate to Naval Engineering: longitudinal direction, transverse direction, athwartships, midships, amidships, draft, mean draft, displacement, resultant weight, buoyant force, centerline, baseline, keel, heel, roll, list, and trim.

17.	Be familiar with the concepts involved in Bernoulli's Theorem.

1.1   Drawings, Sketches, and Graphs

Drawing, sketching, and graphing; are they different? Bananas, oranges, and coconuts are all edible fruits, yet they all have a sharp contrast in taste and texture. But they all must be peeled to be eaten. Clearly there are similarities and differences in bananas, oranges, and coconuts just as there are in drawing, sketching, and graphing.

The common thread in drawing, sketching, and graphing is that they are all forms of visual communication. If each drawing, sketch, or graph could talk, it would be saying something important about the relationship of two or more parameters. Engineering is all about communicating ideas to others, and drawings, sketches, and graphs are three principle methods by which ideas are communicated. It is your job to be able to prepare a drawing, sketch, or graph so that it can effectively communicate information to someone else looking at your work.

1.1.1   Graphs are used to represent the relationships between variables, such as data taken during an experiment. Graphs are also used to represent analytical functions like $y = mx + b$. Graphs require that you use exact coordinates and visually represent relationships between variables in perfect proportions on the paper. A proper graph can be time consuming and require skill to prepare. Computers and spreadsheet programs can be used as tools in effectively preparing a graph.

Graphs are to be done on graph paper (or with a computer) that has major and minor axes in both the vertical and horizontal directions. Major axes are to be subdivided such that they are easy to read and construct. Axis subdivisions should be consistent with the line spacing on the graph paper. Axis subdivisions that require a lot of interpolation and guessing when obtaining data are to be avoided.

Graphs must have a title that describes what is being plotted, and each axis must have a title that thoroughly describes the variable being plotted. Additionally, each axis title must include the symbol for the variable being plotted, and appropriate units for that variable.

When more than one set of data is being plotted, you must clearly identify each set of data. This is best accomplished using a legend, or by individually labeling each curve.

Graphs usually reveal a relationship that may not have been readily apparent. For instance, a graph may show a linear relationship between two variables, or it may show that one variable varies exponentially with respect to the other variable. This relationship may not be apparent when just looking at a list of numbers.

Figure 1.1 on the following page is an example of a properly prepared graph. Note that data points only have been plotted. It would be up to the creator of the plot to fair a curve through the data.

27-B-1 Model Righting Arm Curves
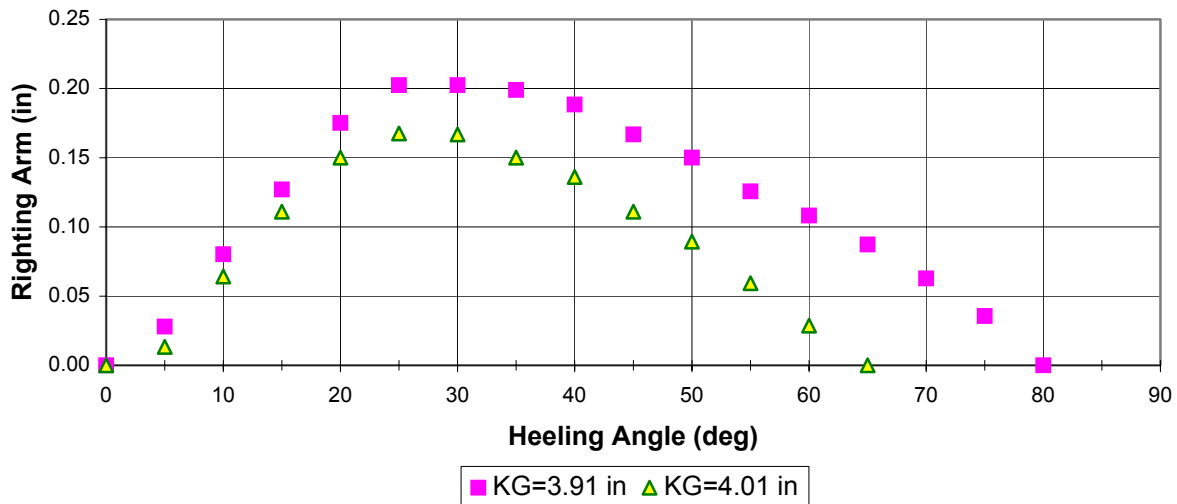**Displacement = 42.97 lb**

Figure 1.1: Graphing example

1.1.2   Drawings are prepared to scale, and are used to show the exact shape of an object, or the relationship between objects so that an object can be manufactured. For example, ship's drawings are used by builders to place a pump within a space, or to route pipes through compartments. Drawings are also used to define the shape of a ship's hull.

1.1.3   Sketches on the other hand are quick and easy pictures of drawings or graphs. The idea behind a sketch is to not show an exact, scale relationship, but to show general relationships between variables or objects. The idea is not to plot out exact points on graph paper but to quickly label each axis and show the general shape of the curve by "free-handing" it.

1.2 Dependent and Independent Variables and Their Relationships

In general, the horizontal axis of a graph is referred to as the *x-axis*, and the vertical axis is the *y-axis*. Conventionally, the x-axis is used for the independent variable and the y-axis is the dependent variable. The "dependent" variable's value will depend on the value of the "independent" variable. An example of an independent variable is time. Time marches on quite independently of other physical properties. So, if we were to plot how an object's velocity varies with time, time would be the independent variable, and velocity would be the dependent variable. There can be, and often is, more than one independent variable in a mathematical relationship.

The concept of a dependent and independent variable is fundamental but extremely important. The relationship of the dependent variable to the independent variable is what is sought in science and engineering. Sometimes you will see the following notation in math and science that lets you know what properties (variables) that another variable depends on, or is a function of.

Parameter Name = f(independent variable #1, independent variable #2, etc)

"Parameter name" is any dependent variable being studied. For example, you will learn that the power required to propel a ship through the water is a function of several variables, including the ship's speed, hull form, and water density. This relationship would be written as:

Resistance = f(velocity, hull form, water density, etc)

There are several ways to develop the relationship between the dependent and independent variable(s). One is by doing an experiment and collecting raw data. The data is plotted as discrete points on some independent axis. Figure 1.1 shows how the righting arm of a model used in lab varies with the angle at which it is heeled. Note that data points have been plotted as individual points. Once plotted, a curve is *faired* through the data. Never just connect the points like a "connect the dots" picture in a children's game book. Nature just doesn't behave in a "connect the dots" fashion. Constructing a graph by connecting the points with straight lines indicates to anyone reviewing your work that you have no idea as to the relationships between variables. Fairing or interpolating a curve through experimental data is an art that requires skill and practice. Computers with the appropriate software (curve fitting program) can make the task of fairing a curve relatively simple, producing an empirical equation for the faired curve through regression analysis.

Besides empirical curve fitting experimental data to arrive at a relationship, a scientist or engineer may go about finding a relationship based on physical laws, theoretical principles, or postulates. For example, if theory states that a ship's resistance will increase exponentially with speed, the engineer will look for data and a relationship between variables that supports the theory.

1.3   The Region Under a Curve and the Slope of a Curve

As discussed previously, the shape of a curve on a graph reveals information about the relationship between the independent and dependent variables. Additionally, more information can be obtained by understanding what the region under the curve and the slope of the plot is telling you.

The region under the curve is also referred to as the area under the curve. The term "area" can be misleading in one sense because this "area" can physically represent any quantity or none at all. Don't be confused or mislead into thinking that the area under the curve always represents area in square feet, as it may not.

In calculus, you integrated many functions as part of the course work. In reality, integration is the task of calculating the area under a curve. Engineers often integrate experimental data to see if a new relationship between the data can be found. Many times this involves checking the units of the area under the curve and seeing if these units have any physical meaning. To find the units of the area under the curve, multiply the units of the variable on the x-axis by the units of the variable on the y-axis. If the area under the curve has any meaning, you will often discover it in this manner. For example, Figure 1.2 shows how the velocity of a ship increases over time.
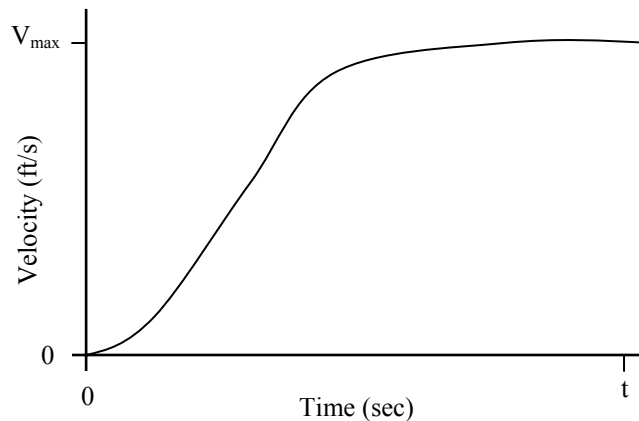

Figure 1.2: Ship's speed as a function of time

To find the area under the curve from a time of zero seconds until time t, integrate the function as shown below:

$$A = \int_{0}^{t} V(t)dt$$

To see if the area under the curve has physical meaning, multiply the units of the x-axis by the units of the y-axis. In this case the x-axis has units of seconds and the y-axis has units of feet per second. Multiplying these together yields units of feet. Therefore, the area under the curve represents a distance; the distance the ship travels in t seconds.

1 - 4

The slope of a curve is the change in the dependent variable over some change in the independent variable. Many times the slope is referred to as the "rise over the run". In calculus, the slope of a function is referred to as the derivative of the function. Just as the area under a curve may have physical meaning, the slope of a curve may also have physical meaning.

The slope of a curve at any point is called the instantaneous slope, since it is the value of the slope at a single instance on the curve. Strict mathematicians may only use the term "instantaneous" when the independent variable is time, as in the slope at a particular instant in time. However, engineers often interpret the slope in more broad terms. To determine if the slope of a curve has physical meaning, divide the units of the y-axis by the units of the x-axis (the rise over the run). For example, look once again at Figure 1.2. The slope of the curve is written in calculus form as:

$$slope = \frac{dV}{dt}, \text{ the change in velocity with respect to time}$$

To analyze the units of the slope, divide the units of velocity, feet per second, by the units of time, seconds. This yields units of feet per second squared, the units of acceleration. Thus, the slope of the curve in Figure 1.2 has meaning, the acceleration of the ship at any point in time.

1.4 Unit Systems

There are three commonly used units systems in engineering. You have probably used each of them at one point or another. Different disciplines in science prefer using different units systems by convention. For example, the unit system of science is the metric system, known as the International System of Units (SI) from the French name, Le Système International d'Units. The SI system is used worldwide in science, engineering, and commerce. Another common system of units is the English "pound force – pound mass" system. This system is commonly used in the fields of thermodynamics and heat transfer. The third unit system is the system we will use for this course: the British gravitational system. The British gravitational system is also known as the "pound – slug" system. The "pound – slug" system of units is used by naval architects and structural engineers; it is also the system of units that we tend to use in our daily lives, whether we realize it or not.

The SI system and the British "pound – slug" systems of units have their roots in Newton's second law of motion: force is equal to the time rate of change of momentum. Newton's second law defines a direct relationship between the four basic physical quantities of mechanics: force, mass, length, and time. The relationship between force and mass is written mathematically as:

$$\vec{F} = m\vec{a}$$

where "*F*" denotes force, "*m*" denotes mass, and "*a*" denotes acceleration,

In the SI system of units, mass, length, and time are taken as basic units, and force has units derived from the basic units of mass, length, and time. Mass has units of kilograms (kg), length has units of meters (m), and time is defined in seconds (s). The corresponding derived unit of

force is the Newton (N). One Newton is defined to be the force required to accelerate a mass of one kilogram at a rate of one meter per second squared (1 m/s^2). Mathematically, this can be written as:

$$F = (1 \text{ kg}) \times (1 \text{ m/s}^2)$$

$$F = 1 \text{ kg-m/s}^2 = 1 \text{ Newton}$$

In the British gravitational (pound –slug) system, the physical quantities of force (lb), length (ft), and time (s) are defined to be the base units. In this system, units of mass are derived from the base units of force, length, and time. The derived unit of mass is called the *slug*. One slug is defined as the mass that will be accelerated at a rate of one foot per second squared (ft/s^2) by one pound of force (lb). Substituting into Equation 1.1 yields:

$$F = ma$$

$$1 \text{ } lb = (mass) \times (1 \text{ } ft/s^2)$$

or, $$mass = \frac{1 lb}{1 \dfrac{ft}{s^2}} = 1\frac{lb-s^2}{ft} = 1 slug$$

To find the weight of an object using the "pound-slug" system, one would use Equation 1.1, substituting the magnitude of the acceleration of gravity as the acceleration term. In the "pound-slug" system, the acceleration of gravity (*g*) is equal to 32.17 ft/s^2.

Example: An object has a mass of 1 slug. Calculate its corresponding weight.

$$Weight = (mass) \times (acceleration)$$

$$Weight = (1 \text{ slug}) \times (32.17 \text{ ft/s}^2) = (1 \text{ lb-s}^2/\text{ft}) \times (32.17 \text{ ft/s}^2)$$

$$Weight = 32.17 \text{ lb}$$

Table 1.1 is a listing of some of the common physical properties used in this course and their corresponding units in the British gravitational system.

Property	Length	Time	Force	Mass	Density	Pressure
Units	foot (ft)	seconds (s)	pounds (lb)	slug	slug/ft^3	lb/in^2
			or	or	or	
			long ton (LT)	lb-s^2/ft	lb-s^2/ft^4	

Table 1.1: Common physical properties and their corresponding units.

1.4.1   Unit Analysis

Unit analysis is a useful tool to help you solve a problem. Unit analysis is nothing more than ensuring that the units that correspond to numeric values in an equation produce units that correspond to the property you are solving for. For example, if a problem calls for you to find the weight of an object, you should know that you want your final result to be in pounds or long tons.

The best method for conducting unit analysis is to use the "ruled lines" method. Using this method divides numeric values for variables and their units into an organized equation that is easy to follow.

Example problem using the "ruled lines" method

A ship floating in salt water has a displaced volume of 4,000 ft^3. Calculate the ship's weight in long tons.

As you will find out in the next chapter, a ship's weight is equal to the weight of the volume of water displaced by the ship (Archimedes Principle). This is written mathematically as:

Displacement = (*water density*) × (*acceleration of gravity*) × (*volume*)

or, $\Delta = \rho g \nabla$

$\rho = 1.99$ lb-s^2/ft^4
$g = 32.17$ ft/s^2
1 LT = 2240 lb

$$\Delta = \left[\frac{1.99 lb - s^2}{ft^4} \right] \times \left[\frac{32.17 ft}{s^2} \right] \times \left[4,000 ft^3 \right] \times \left[\frac{LT}{2240 lb} \right]$$

$\Delta = 114.32$ LT

Note how the applicable equation was written in symbolic form, and then rewritten numerically with numeric values for each variable and appropriate units written in the same order as the general equation. Also note that in the numerical equation units cancel themselves until the desired units of long tons are obtained.

Notice how neat and organized this approach makes the calculation. This makes it very easy for someone else reviewing your work to understand what you did and then verify your work. Please do all calculations for this course in this manner. It is a wonderful engineering practice that effectively communicates your ideas to others. You will have plenty of opportunity throughout your career to communicate ideas, and this method is highly desirable when presenting calculations to higher authority.

Lastly, check your final answer for reasonability in magnitude and for proper units. You should have a "ball park" idea on the magnitude of the final answer. If you get a seemingly outrageous result, state that "this result is unreasonable" on the paper. Part of engineering is being able to recognized when a result doesn't make sense. For instance, if you are trying to calculate the final draft of a YP after placing a pallet of sodas on the main deck, and you end up with a final draft of 35 feet (a YP's normal draft is approximately 6 feet), you should recognize that the result of 35 feet makes no physical sense (the YP would have sunk). Also, watch the units in the final answer. For example, if you are calculating a volume (cubic feet) but unit analysis shows pounds, you should recognize that you've made a mistake. When grading homework and exams, units are a dead giveaway when it comes to finding mistakes in your calculations. A quick verification of units can save you many points on an exam. Analogy: if you are trying to find corruption in business, follow the money. If you are trying to find errors in engineering, follow the units.

1.5   Rules for Significant Figures

In this course you will perform calculations involving values obtained from a variety of sources, including graphs, tables, and laboratory data. The accuracy of these numbers gives rise to the subject of significant figures and how to use them. Numbers that are obtained from measurements contain a fixed number of reliable digits called significant figures. The number "13.56" has four significant figures.

There are two types of numbers used in engineering calculations. The first type of number is called an *exact number*. An exact number is one that comes from a direct count of objects, or that results from definitions. For example, if you were to count 5 oranges, there would be exactly 5 oranges. By definition, there are 2,240 pounds per long ton. Exact numbers are considered to possess an infinite number of significant figures.

The second type of numbers used in engineering calculations is a *measurement*. The number of significant figures used in measurements depends on the precision or accuracy of the measuring device. A micrometer capable of measuring an object to the fourth decimal place is much more accurate than a ruler marked off in 1/8 inch increments. The more precise the measuring device, the more significant figures you can use when reporting your results.

When combining several different measurements, the results must be combined through arithmetic calculations to arrive at some desired final answer. To have an idea of how reliable the calculated answer is we need to have a way of being sure the answer reflects the precision of the original measurements. Here are a few simple rules to follow:

> For addition and subtraction of measurement values, the answer should have the same number of decimal places as the quantity having the least number of decimal places. For example, the answer in the following expression has only one decimal place because "125.2" is the number with the least number of decimal places and it only contains one.

$$3.247 + 41.36 + 125.2 = 169.8$$

Many people prefer to add the original numbers and then round the answer. If we enter the original numbers in the calculator, we obtain the sum "169.807". Rounding to the nearest tenth (one decimal place) again gives the answer "169.8"

For multiplication and division, the number of significant figures in the answer should not be greater than the number of significant figures in the least precise factor. For example, the answer in the following expression has only two significant figures because the least precise factor "0.64" contains only two.

$$\frac{3.14 \times 2.751}{0.64} = 13.$$

When using exact numbers in calculations, we can forget about them as far as significant figures are concerned. When you have mixed numbers from measurements with exact numbers (common in labs) you can determine the number of significant figures in the answer the usual way, but take into account only those numbers that arise from measurements.

For convenience in this course, you can assume all numbers are exact unless otherwise told. Give a reasonable number of decimal places in your answer (two or three are usually sufficient) and use some common sense. For example, if someone asked how much you weigh you wouldn't say 175.2398573 pounds. As a general rule give at least two numbers after the decimal or as many as it takes to show your instructor your answer reflects you have done everything correctly.

Some comments on computational accuracy. Your calculator is a tool, enabling you to be much more accurate in calculating numbers. Use your tools wisely. When confronted with the task of multiplying a whole number by a fraction such as 2/3, many students will write out and input a value of "0.66" into their calculator instead of "2/3". There is a huge difference in these two values. The factor "2/3" is much more accurate than "0.66" and will yield a more accurate result. The same thing applies with the value of "pi". Let the calculator carry "pi" out to however many decimal places it wants rather than use the grade school value of "3.14".

1.6   Linear Interpolation

Engineers are often faced with having to interpolate values out of a table. For example, you may need the density of fresh water at a temperature of 62.3°F in order to solve a problem, however, tabulated data gives you densities at 62°F and 63°F. Finding the density of water at 62.3°F requires interpolation. The most common assumption in interpolation is that parameters vary in linear fashion between the listed parameters. This means that you can approximate the curve between values in the table with a straight line and easily determine the desired value. The following example shows how easy this is.

Example: Given the following values of the density of fresh water, find its density at a temperature of 62.7°F.

Temperature (°F)	Density (lb-s^2/ft^4)
61	1.9381
62	1.9379
63	1.9377

Solve for the density at 62.7°F as follows:

$$\frac{62.7°F - 62°F}{63°F - 62°F} = \frac{\rho_{62.7°F} - \rho_{62°F}}{\rho_{63°F} - \rho_{62°F}}$$

The algebraic solution for density at 62.7°F yields:

$$\rho_{62.7°F} = \rho_{62°F} + \left(\frac{62.7°F - 62°F}{63°F - 62°F}\right)(\rho_{63°F} - \rho_{62°F})$$

Doing the numerical substitution and solving,

$$\rho_{62.7°F} = 1.9739\frac{lb - s^2}{ft^4} + \left(\frac{62.7 - 62}{63 - 62}\right)\left(1.9377\frac{lb - s^2}{ft^4} - 1.9379\frac{lb - s^2}{ft^4}\right)$$

$$\rho_{62.7°F} = 1.93776\frac{lb - s^2}{ft^4}$$

When doing this (or any) type of calculation on a test, quiz, homework, or lab, you must show the general equation, then substitution of numerical values. That way another person reviewing your work can understand your thought process. When interpolating, always check your result and make sure it is between the other values in the table.

1.7   A Physics Review

The science of physics is the foundation of engineering. Therefore, we will undertake a short review of some concepts of physics prior to commencing our study of naval engineering. This review of physics will touch on the basics of scalars, vectors, moments, and couples. In addition, we will also take a look at the basics of statics.

1.7.1   Scalars and Vectors

A scalar is a quantity that only has magnitude. Mass, speed, work, and energy are examples of scalar quantities. A vector, on the other hand, is expressed in terms of both a magnitude and direction. Common vector quantities include velocity, acceleration, and force. Engineers denote vectors by placing a small arrow over the vector quantity. For example, a force vector would be written as $\vec{F}$. One other method to denote a vector, especially in textbooks, is to show a vector quantity in boldface type; such as **F**.

As mentioned above, a vector quantity has both a magnitude and direction, and they obey the parallelogram law of vector addition. Figure 1.3 shows the how vectors **a**, **b**, and **c** are related.
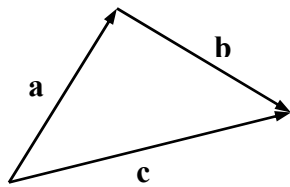
$$\mathbf{a} + \mathbf{b} = \mathbf{c}$$

$$\mathbf{c} - \mathbf{a} = \mathbf{b}$$
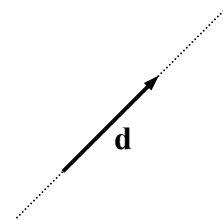
Figure 1.3: Vector addition   Figure 1.4: Vector line of action

The line of action of a vector is an imaginary line running coincident with the vector extending to infinity in both directions along the line, as shown by vector **d** in Figure 1.4.

Many times it is useful to resolve vectors into components in each principle direction. In the rectangular coordinate system, the principle directions are the x, y, and z directions as shown below.
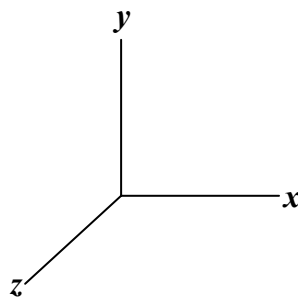
Figure 1.5: Rectangular coordinate system

Consider the vector, **F**, in the *x-y* plane as shown below in Figure 1.6. **F** can be resolved into components in both the *x* and *y* direction, **F**$_x$ and **F**$_y$. The scalar magnitudes of **F**$_x$ and **F**$_y$ can be found using the relationships shown in Figure 1.6.
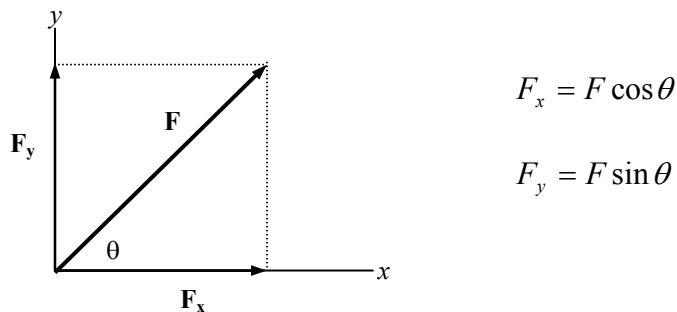
$$F_x = F \cos\theta$$

$$F_y = F \sin\theta$$

Figure 1.6: Resolution of a vector into its components

In engineering diagrams, vectors are shown as arrows. The length of the arrow, from tail to head should represent the magnitude of the vector (the larger the magnitude, the longer the vector), and the arrow's direction represents the direction in which the vector acts. The exact placement of the vector is important in engineering. Either the head or tail of a vector may be placed at its point of application.

1.7.2 Forces

In this course we will use two types of forces: a point force and a distributed force. A point force is a force that has a single point of application. An example of a point force would be a person standing in the middle of a bridge; the weight of the person is considered to act at a single point of the bridge. A distributed force is a force that acts over a distance or an area. An example of a distributed force would be a pile of gravel that is dumped onto the bridge. Figure 1.7 shows the difference between a point force and a distributed force. A force causes an object to move in the direction of the force's line of action. This motion is also called translation.

A distributed force can be resolved into a single, point force acting at the centroid of the distributed force. Figure 1.7 shows a distributed force of 100 lb/ft acting over a distance of 10 ft. The resultant, point force would be found by multiplying the magnitude of the distributed force by the distance over which it acts, in this case yielding a resultant point force of 1000 lb. This point force then acts at the center of the distributed force, in this case 20 ft from the end of the bridge.
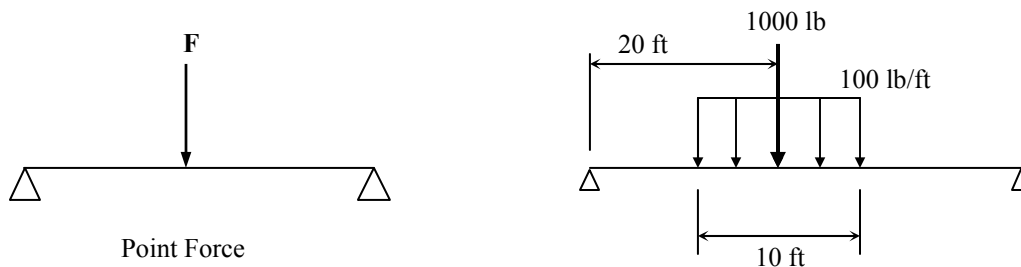
Distributed force and its resultant point force

Figure 1.7: Point forces and distributed forces

1.7.2.1 Force Versus Weight

When you know the mass and acceleration of an object, you can use Newton's second law to calculate the force exerted on the object. When an object is near the earth's surface it will experience a gravitational force due to the acceleration of gravity acting on the object's mass. This gravitational force is known as the object's weight. Therefore, weight is also a force, and therefore a vector quantity. A vector representing an object's weight **always** acts towards the center of the earth, and is drawn vertically with the vector's head pointing down. The weight of an object is represented with units of pounds, or for most of this course, units of long tons (LT). The weight (or displacement) of a ship is always given in long tons (1 LT = 2240 lb), and is represented by the Greek letter delta, Δ_S.

1.7.3   Moments Created by Forces

A moment is created by the action of a force applied at some distance from a reference point, such that the line of action of the force does not pass through the reference point. Whereas a force causes translation of an object, application of a moment creates the tendency for an object to rotate. Additionally, the force creating the moment can also cause translation of the object.

A common example of an applied moment is when you tighten a nut with a wrench. A force is applied to the end of the wrench opposite the nut, causing the wrench handle to move and the nut to rotate. The force acting on the wrench handle creates a moment about the nut.

The magnitude of a moment is the product of the magnitude of the force applied and the distance between a reference point and the force vector. Moments have units of force multiplied by distance: foot-pounds (ft-lb) or foot-long tons (ft-LT). Mathematically, the magnitude of a moment is written as:

$$Moment = Force \times Distance$$
$$M = F \times d$$

Figure 1.8 shows a wrench turning a nut. The wrench handle has length l, with a force of magnitude F acting at the end of the wrench. Force F creates a moment about the nut, causing the nut to rotate. The magnitude of the moment about the nut is $F \times l$. Note that the magnitude of the moment will increase if either the force is increased, or the length of the wrench handle is increased.
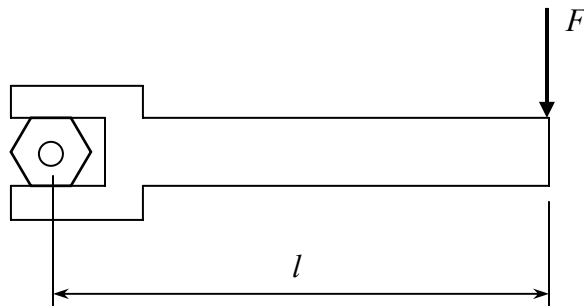

Figure 1.8: Example of a moment applied with a wrench

1.7.4   Couples

A couple is a special type of moment that causes pure rotation without translation. A couple is formed by a pair of forces, equal in magnitude, and acting parallel to each other but in opposite directions, separated by some distance. Figure 1.9 shows two forces creating a couple.
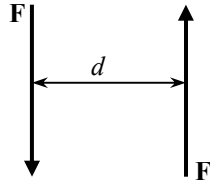

Figure 1.9: Two forces creating a couple

The magnitude of a couple is calculated by multiplying the magnitude of **one** force by the distance separating the two forces. In Figure 1.9, the magnitude of the couple would be:

$$Couple = Force \times distance$$

$$C = F \times d$$

Just as moments have units of foot-pounds, couples also have units of foot-pounds.

1.7.5   Static Equilibrium

When one or more forces is acting on an object, the sum of forces in the x, y, and z directions will ultimately tell you if the object will translate or rotate. If the sum of forces and/or moments does not equal zero, the object will translate or rotate. If the sum of all forces equals zero, and the sum of all moments equals zero, the object will neither translate nor rotate, a condition referred to as "static equilibrium". The two necessary and sufficient conditions for static equilibrium are:

    1.   The sum (resultant) of all forces acting on a body is equal to zero.

    2.   The sum (resultant) of all moments acting on a body is equal to zero.

Mathematically, this is written as:

$$\sum \vec{F} = 0 \qquad\qquad \sum \vec{M} = 0$$

The analysis of an object in static equilibrium is also known as the study of "statics". The analysis of ships in static equilibrium is the study of hydrostatics. This course will use the principle of static equilibrium to explain the effects of list and trim on a ship, as well as describing the behavior of materials under different loading conditions.
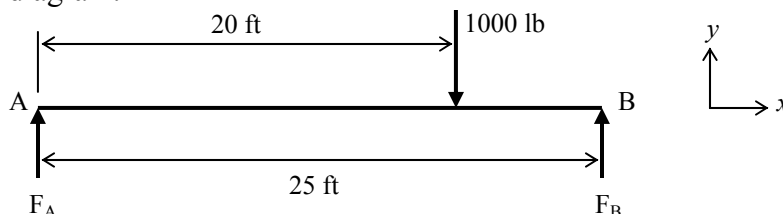
The principles of static equilibrium are very useful when solving for unknown forces acting on a body, as demonstrated in the following example problem.

Example:

    A beam 25 ft in length is supported at both ends as shown below. One end of the beam is pinned (fixed) in place, and rollers support the other end. A force of 1000 lb is applied 20 ft from end "A". Calculate the vertical reaction forces exerted on the beam by the supports.
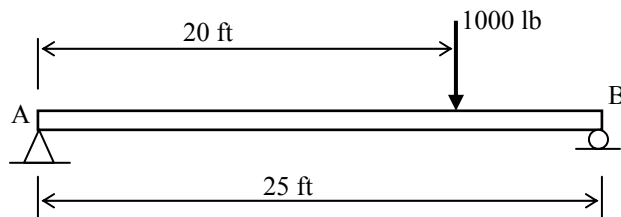
    Re-drawing the beam with reaction forces at points "A" and "B", gives the following "free body diagram."

To solve for the reaction forces, F_A and F_B, we will use the principles of static equilibrium. Summing forces in the vertical (y) direction and setting the sum equal to zero yields:

$$\sum F_y = 0$$

$$F_A + F_B - 1000lb = 0 \qquad\qquad \text{equation (1)}$$

Summing moments about point "A" and setting the sum equal to zero yields:

$$\sum M_A = 0$$

$$(1000 \ lb)(20 \ ft) - (F_B)(25 \ ft) = 0$$

Solving for the reaction force, F_B yields:

$$F_B = \frac{(1000lb)(20 \ ft)}{25 \ ft} = 800lb \qquad \text{the reaction force at "B" equals } 800lb$$

Substituting F_B into equation (1) and solving for F_A yields:

$$F_A = 1000lb - F_B = 1000lb - 800lb$$

$$F_A = 200 \ lb \qquad\qquad \text{the reaction force at "A" equals } 200lb$$

Reviewing the methodology for problem solving:

1. Write down a problem statement.
2. Draw a diagram that shows the relationship between all variables.
3. Write general equations (no numbers at this point) that govern the problem.
4. Algebraically solve the equations for the desired final value.
5. Make numerical substitutions and solve your final answer.

Finally, look at your answer(s) and determine if it makes sense. If your answer doesn't make physical sense, you've probably made a mistake somewhere.

1.7.6 Pressure and Hydrostatic Pressure

On the most fundamental level, pressure is the effect of molecules colliding. All fluids (liquids and gases), have molecules moving about each other in a random chaotic manner. These molecules will collide and change direction exerting an impulse. The sum of all the impulses produces a distributed force over an area which we call pressure. Pressure has units of pounds per square inch (psi).

There are actually three kinds of pressure used in fluid dynamics. They are static, dynamic, and total pressure. The pressure described in the first paragraph is static pressure. Dynamic pressure is the pressure measured in the face of a moving fluid. Total pressure is the sum of static and dynamic pressure. If a fluid is assumed to be at rest, dynamic pressure is equal to zero. The pressures we will be using in this course will be based on fluids being at rest.

Consider a small object sitting in water at some depth, z. If the object and water are at rest (velocity equals zero), then the sum of all forces acting on the object is zero, and the dynamic pressure acting on the object is zero. The pressure that the water exerts on the object is static pressure only; in the analysis of fluids, this static pressure is referred to as "hydrostatic pressure." Hydrostatic pressure is made up by distributed forces that act normal to the surface of the object in the water. These forces are referred to as "hydrostatic forces". The hydrostatic forces can be resolved into horizontal and vertical components. Since the object is in static equilibrium, the horizontal components of hydrostatic force must sum to zero and cancel each other out. For the object to remain at rest, the vertical component of hydrostatic force must equal the weight of the column of water directly above the object. The weight of the column of water is determined using the equation:

Weight = (*density*) × (*gravity*) × (*volume of water*)

$W = \rho g V = \rho g A z$

where: ρ = water density (lb-s^2/ft^4)
g = acceleration of gravity (ft/s^2)
A = surface area of the object (ft^2)
z = depth of the object below the water's surface (ft)

Therefore, the hydrostatic force acting on an object at depth, z is equal to:

$F_{hyd} = \rho g A z$

Dividing hydrostatic force by the area over which it acts yields the hydrostatic pressure acting on the object:

$P_{hyd} = \rho g z$ $\Rightarrow$ $F_{hyd} = P_{hyd} \times Area$

This equation can be used to find the hydrostatic pressure acting on any object at any depth. Note that hydrostatic pressure varies linearly with the depth of the object.

1.7.7 Mathematical Moments

In science and math, the following integrals appear in the mathematical descriptions of physical processes:

$$\int s\,dm \qquad\qquad \int s^2\,dm$$

Where "s" typically represents a distance and "dm" is any differential property "m". Because these integrals are familiar across many disciplines of science and engineering, they are given special names, specifically the first moment and second moment. The order of the moment is the same as the exponent of the distance variable "s". Common properties of "m", as used in Naval Engineering are length, area, volume, mass, and force (weight).

Some common mathematical moments used in this course, along with their physical meanings and uses, are given below.

First Moment of Area

Consider the region shown in Figure 1.10. The region has a total area, "A" and is divided into differential areas "dA". The moment of each dA about an axis is the distance of each dA from the axis multiplied by the area dA. The total first moment of area about an axis can be found by summing all the individual moments of area through the integral process.
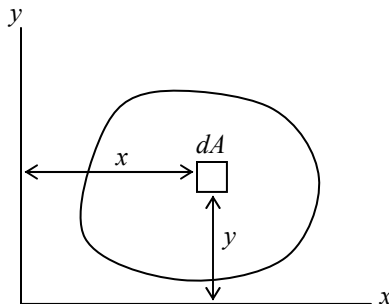

Figure 1.10: Diagram used for moment of area calculations

The first moment of area about the x-axis is found using the following equation:

$$M_x = \int y\,dA$$

The first moment of area about the y-axis is found using:

$$M_y = \int x\,dA$$

Note that the first moment of area will have units of distance cubed (i.e. ft^3). The first moment of area will be used in the calculation of the region's centroid.

Geometric Centroids

The centroid of an object is its geometric center. To calculate the centroid of an area, consider the region shown below in Figure 1.11. The region has total area A_T, and is subdivided into small areas of equal size, A_n, located a distance x_n from the x-axis and y_n from the y-axis. The centroid of the region as measured from the y-axis is denoted "*x-bar*", and as measured from the x-axis is denoted "*y-bar*".
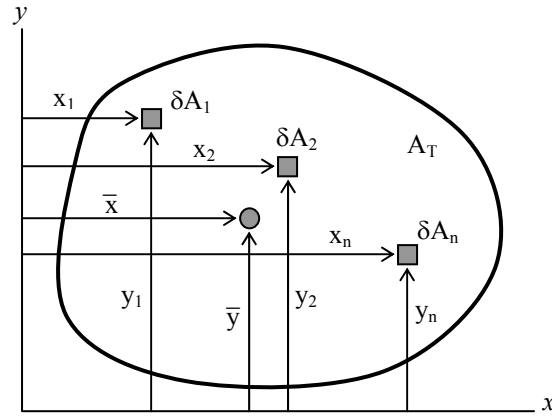

Figure 1.11: Diagram for the calculation of a geometric centroid

The centroid of an object can be thought of as the average distance that all A's are from each axis. This is determined by calculating the first moment of area of each A, summing each individual first moment, and then dividing by the total area of region A. Mathematically, the centroid of the region from each axis is written as:

$$\bar{y} = \frac{y_1 \delta A_1 + y_2 \delta A_2 + ... + y_n \delta A_n}{\delta A_1 + \delta A_2 + ... + \delta A_n} = \frac{\sum_{i=1}^{n} y_i \delta A_i}{\sum_{i=1}^{n} \delta A_i} = \frac{\sum_{i=1}^{n} y_i \delta A_i}{A_T}$$

$$\bar{x} = \frac{x_1 \delta A_1 + x_2 \delta A_2 + ... + x_n \delta A_n}{\delta A_1 + \delta A_2 + ... + \delta A_n} = \frac{\sum_{i=1}^{n} x_i \delta A_i}{\sum_{i=1}^{n} \delta A_i} = \frac{\sum_{i=1}^{n} x_i \delta A_i}{A_T}$$

where: y_i = vertical distance of each δA from the x-axis (ft)
x_i = horizontal distance of each δA from the y-axis (ft)
A_T = total area of the region (ft^2)

The previous relationships can also be written as calculus equations. As each element A becomes small enough, it becomes a differential area, dA, and the distance of each element from its axis becomes x or y instead of x_i and y_i. The equation for the centroid can then be written as:

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int y\,dA}{A_T} \qquad \text{and,} \qquad \bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int x\,dA}{A_T}$$

Many times in engineering, we are confronted with calculating the centroid of regular geometric shapes. Calculation of the centroid of a regular geometric shape can best be illustrated using the shape consisting of a rectangle and right triangle as shown below in Figure 1.12.
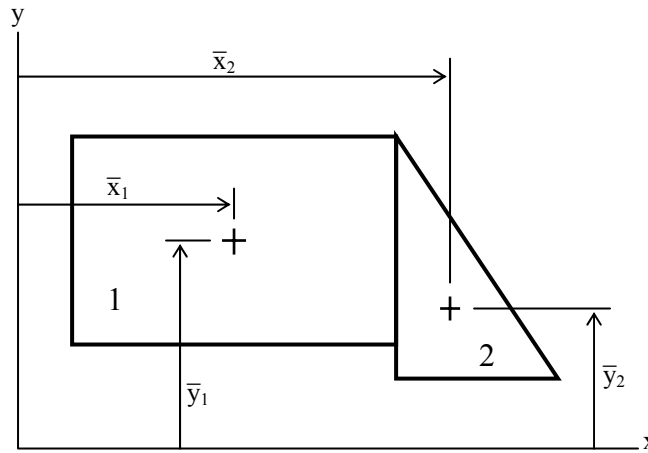

Figure 1.12: Diagram for calculating the centroid of a regular geometric shape

The region in Figure 1.12 is divided into two known geometric areas, rectangle 1 and right triangle 2. The centroid of the total region is found as shown below:

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2} \qquad \text{and,} \qquad \bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2}$$

> where: $\bar{x}_1$ and $\bar{x}_2$ are the x distances to the centroid of areas 1 and 2 respectively
>
> A_1 and A_2 are the areas of 1 and 2 respectively

This procedure can be used to find the centroid of any regular geometric shape by using the following equations:

$$\bar{x} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \text{and,} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \bar{y}_i A_i}{\sum_{i=1}^{n} A_i}$$

Second Moment of Area

The second moment of area (I) is a quantity that is commonly used in naval architecture and structural engineering. The second moment of area is used in calculations of stiffness. The larger the value of the second moment of area, the less likely an object is to deform. The second moment of area is calculated using the following equation:

$$I = \int s^2 dA$$ where s represents a distance from an axis, and dA represents a differential area, and I denotes the second moment of area
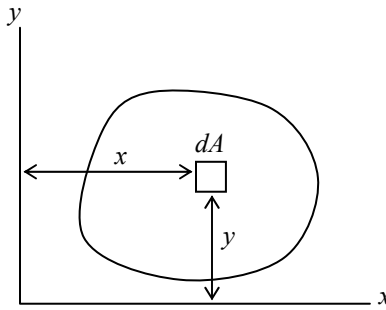
Figure 1.13: Diagram for the determination of the second moment of area

Using Figure 1.13, the second moment of area of the region about the x-axis can be found using the following equation:

$$I_x = \int y^2 dA$$

Likewise, the second moment of area of the region about the y-axis can be found using:

$$I_y = \int x^2 dA$$

Note that the second moment of area will have units of length to the fourth power (in^4 or ft^4).

Engineering calculations commonly require the second moment of area to be calculated about an object's centroidal axes. To make life easier for engineers, many texts and reference manuals provide equations for the second moment of area of many shapes about their centroidal axes. Appendix C contains equations for the second moment of area and other properties of some common geometric shapes.

Parallel Axis Theorem

Many times there exists a requirement that the second moment of area of an object be calculated about an axis that does not pass through the object's centroid. This calculation is easily accomplished using the parallel axis theorem. The parallel axis theorem can best be illustrated using Figure 1.14 below. The region shown has area A, and has its centroid located at point "C" with axes x_c and y_c passing through the centroid of the region, and parallel to the x and y axes respectively. The centroid of the region is located a distance d_1 from the x-axis, and distance d_2 from the y-axis.
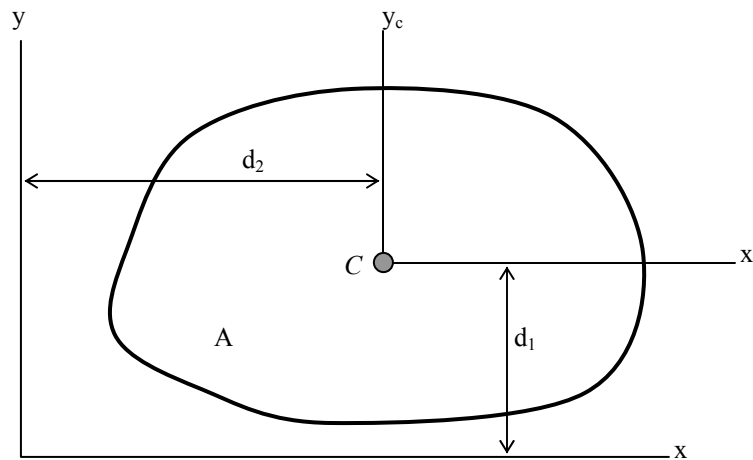

Figure 1.14: Diagram for calculating the second moment of area

The second moment of area of the region is calculated using the following equations. The derivation of these equations is not presented here, but can be found in any statics or strength of materials text.

Second moment of area about the x-axis:

$$I_x = I_{xc} + Ad_1^{\,2}$$

where: $I_x =$ second moment of area about the x-axis
       $I_{xc} =$ second moment of area of the region about the x-axis passing through the centroid of the region
       $A =$ area of the region
       $d_1 =$ distance of the region's centroidal x-axis from the x-axis

Similarly, the second moment of area of the region about the y-axis can be found using:

$$I_y = I_{yc} + Ad_2^{\,2}$$

What follows is an example for calculating the second moment of area of an object using the parallel axis theorem.

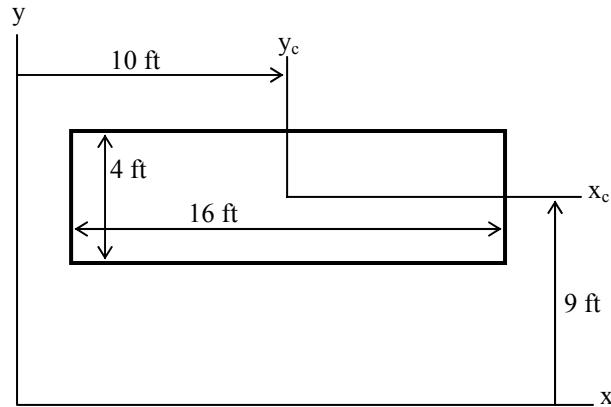**Example:** Calculate the second moment of area of the rectangle about the x-axis.

Figure 1.15: Illustrating the parallel axis theorem

The second moment of area of the rectangle about the x-axis is found using the parallel axis theorem:

$$I_x = I_{xc} + Ad_1^{\,2}$$

where: I_{xc} = second moment of area of the rectangle about its centroidal axis
  A = area of the rectangle
  d_1 = distance of the x-axis from the rectangle's centroidal axis parallel to the x-axis.

$$A = (16\,ft)(4\,ft) = 64\,ft^2$$

From Appendix I, the second moment of area of a rectangle about axis x_c is:

$$I_{xc} = \frac{bh^3}{12} = \frac{(16\,ft)(4\,ft)^3}{12} = 85.33\,ft^4$$

Now apply the parallel axis theorem to find the second moment of area of the rectangle about the x-axis:

$$I_x = (85.33\,ft^4) + (64\,ft^2)(9\,ft)^2 = 5269.33\,ft^4$$

1 - 23

1.8   Definition of a Ship's Axes and Degrees of Freedom

A ship floating freely in water is subject to six degrees of freedom (exactly the same as an airplane flying in the sky). There are three translational and three rotational degrees of freedom associated with a ship. As a quick review, translation refers to motion in a straight line and rotation refers to rotating or spinning about an axis.

In naval architecture, the longitudinal axis of a ship (meaning from bow to stern or stern to bow) is always defined as the *x-axis*. Motion in the longitudinal direction is referred to as *surge*. The x-axis of a ship is usually defined as the ship's centerline with its origin at amidships and the keel. Therefore, positive longitudinal measurements are forward of amidships and negative longitudinal measurements are aft of amidships. Rotation about the x-axis is referred to as *roll*. Figure 1.16 shows the 6 degrees of freedom of a ship. There are also two static conditions of rotation about the x-axis. These are *list* and *heel*. List is a condition produced by a weight shift on the ship. Heel is a condition produced by an external force such as the wind.

A ship's y-axis is used for measurements in the transverse or athwartships (port and starboard) direction. The y-axis has its origin at the keel and on the centerline. Naval architecture convention states that the positive y-direction is starboard of the centerline and that negative measurements are port of centerline. Translational motion in the y-direction is referred to as *sway*, and rotational motion about the y-axis is *pitch*. The static condition of rotation about the y-axis is referred to as *trim*. Similar to list, trim is a condition produced by a weight shift on the ship.

Vertical measurements in a ship are referenced to the keel. Therefore a ship's vertical or z-axis has its origin at the keel and centerline. For most ships, the keel is also the *baseline* for vertical measurements. Translational motion in the z-direction is referred to as *heave*, and rotational motion about the z-axis is *yaw*.

As seen in the above paragraphs, the customary origin for a ship's coordinate system is on the centerline, at the keel and amidships. Sometimes, for ease of computation, the longitudinal origin may be placed at the bow or stern.
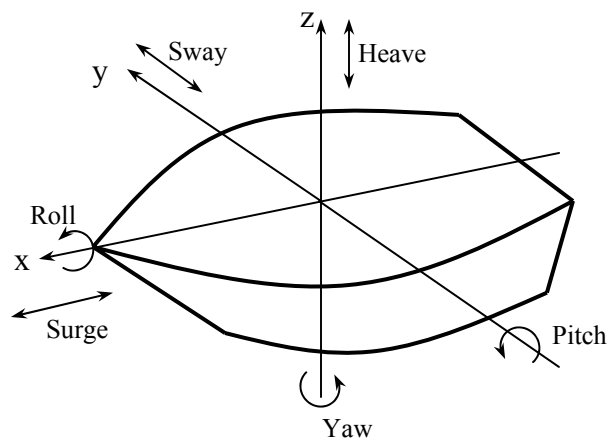

Figure 1.16: A ship's axes and degrees of freedom

1.9 Bernoulli's Equation

It is useful to have a short discussion of external fluid flow. This is the situation that occurs when water flows around a ship's rudder, submarine planes, hydrofoils, or the hull of any vessel moving through the water. The study and analysis of fluid flow is a complex subject, a subject that will baffle researchers for many years to come. However, as students of naval engineering, you should have some basic knowledge of external fluid flow.

Consider a fluid flowing at some velocity, V. Now, think of a line of fluid molecules that are moving in a direction tangent to the fluid's velocity. This line of movement is referred to as a streamline. One method of flow analysis is to consider the fluid to be made up of many streamlines, each layered on top of the other. We will be looking at a group of fluid molecules traveling along one streamline. To further our analysis of the fluid, the fluid flow is assumed to be incompressible, meaning that its density is not changing anywhere along the flow, and that there is no contraction or dilation of the fluid molecules. If the water molecules are not rotating, the flow is called irrotational and the fluid is said to have a vorticity of zero. If there are no shearing stresses between layers of the flow, the fluid is said to be inviscid. Finally, if we assume that a fluid's properties at one point on the streamline do not change with time (although the fluid's properties can change from location to location along the streamline), the fluid flow is said to be steady. In this course, our analysis will assume steady incompressible inviscid flow.

For steady incompressible inviscid flow, the sum of the flow work plus the kinetic energy plus the potential energy is constant along a streamline. This is Bernoulli's Equation and it can be applied to two different points along a streamline to yield the following equation:

$$\frac{p_1}{\rho} + \frac{V_1^{\,2}}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^{\,2}}{2} + gz_2 = \text{constant}$$

where: $p =$ hydrostatic pressure at any point on the streamline
$V =$ the speed of the fluid at any point on the streamline
$g =$ acceleration of gravity
$z =$ height of the streamline above some reference datum
$\rho =$ fluid density

The "p/ρ" term in the equation is the flow work at any point on the streamline, the "$V^2/2$" term is the fluid's kinetic energy at any point on the streamline, and the "gz" term is the fluid's potential energy at any point on the streamline.

Using Bernoulli's equation, we can explain why lift and thrust is generated by flow over hydrofoils, rudders, submarine planes, and propeller blades. We can also describe the pressure distribution of fluid flowing around a ship's hull.

HOMEWORK - CHAPTER ONE

1. a. Sketch the velocity of a Yard Patrol Craft (YP) moving from zero ft/s to its top speed of 20 ft/s. Time to reach maximum speed is 26 seconds. Use velocity as the dependant variable and time as the independent variable.

    b. Give the calculus equation that represents the area under the sketched curve between zero and 10 seconds. What does this area physically represent?

    c. How would you calculate the acceleration of the YP at t = 5 seconds?

2. What does the area under a plot of "ship's sectional area" versus "longitudinal distance" physically represent? State the units of each axis and the area under the curve.

3. Which system of units is used in Naval Engineering I (EN200)? State the units in this system for the following parameters:

    Force   Mass   Volume   Time   Density

    Moment   Couple   Pressure   Second moment of area.

4. A ship's weight (displacement) varies with its draft as shown in the table below.

| Draft (ft)	Weight (LT)
0	0
2	520
4	1050
6	1480
8	2000
10	2420
12	3000

    a. Plot the ship's weight as a function of draft.
    b. Fit a linear curve to the data and calculate the slope.
    c. What does the slope physically represent?
    d. What is the ship's weight at a draft of 9.3 ft?

5.   The deflection of a spring varies with the force applied to the spring as shown in the following experimental data:

| Force (lb)	Deflection (inches)
0	0
1	0.08
2	0.17
3	0.20
4	0.27
5	0.34
6	0.43
7	0.50

    a.   Plot the spring's deflection as a function of force and determine the slope of the resulting curve.
    b.   What physical property does the slope of the data represent?
    c.   From your plotted data, determine how much the spring will deflect if a force of 4.6 pounds is applied.
    d.   Write an equation representing the curve you fit to the data

6.   An oiler transfers 100,000 gallons of F-76 fuel to the tanks of a frigate. The density of the fuel is 1.616 lb-s^2/ft^4.

    a.   The weight of a volume of fluid can be found by the equation $W = \rho g V$, where g represents the acceleration of gravity and V represents the volume. Calculate how many long tons of fuel were transferred from the oiler to the frigate.

    b.   If the oiler's draft changes 1 inch for every 136 LT added or removed, how many feet will the oiler's draft change after transferring the fuel?

    c.   If the frigate's draft changes 1 inch for every 33 LT added or removed, how many inches will the frigate's draft change after receiving the fuel?

7.   A force of 250 lb is acting at an angle of 30 degrees to the x-axis. Determine the x and y components of the 250 lb force.

8.   What are the necessary and sufficient conditions for static equilibrium?

9.   A 1000 lb weight is placed on a beam cantilevered from a wall as shown below. What is the magnitude of the moment that the weight exerts at the wall?

1000 lb

13 ft
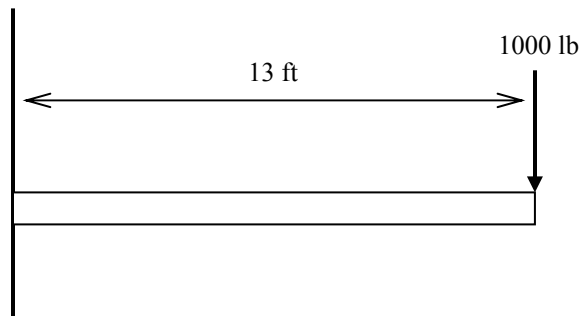
10.   A force of 750 lb is applied to the linkage arm as shown below. The arm pivots about point "P".

a.   A second force is to be applied at point "A". What is necessary magnitude and direction of this force such that a couple is created about "P"?
b.   What is the magnitude of the resulting couple about "P"?

750 lb

5 in   5 in

P   A
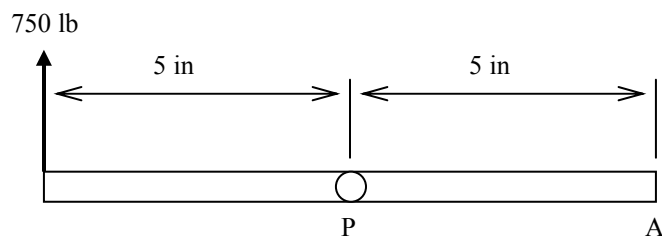
12.   A 285 lb football player is sitting on a see-saw as shown below. Where must a 95 lb gymnast sit in order to balance the see-saw? What is the force at point "P"?

285 lb   3 ft

10 ft   P   10 ft
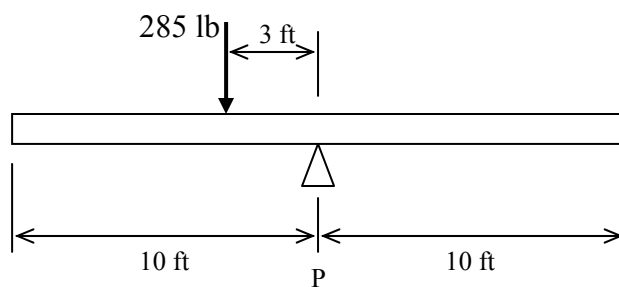
13. A simply supported beam 50 feet in length is loaded as shown below. Calculate the vertical reaction forces at each end of the beam.
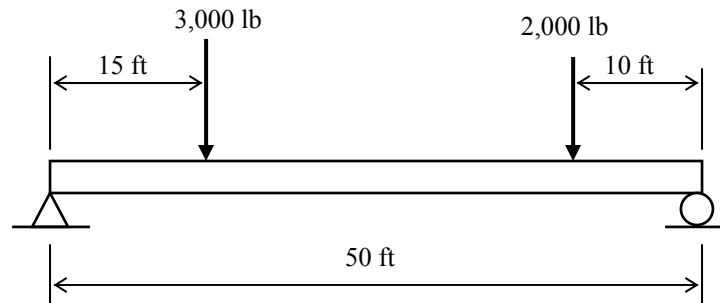
3,000 lb

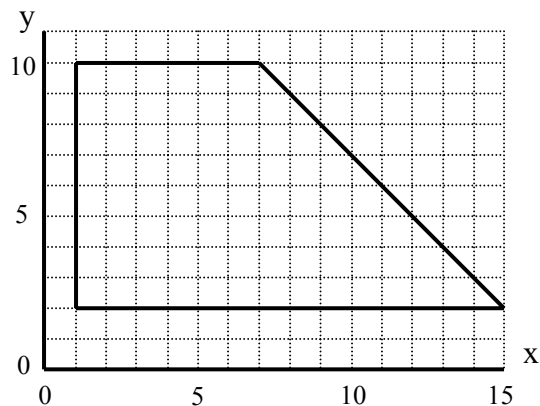2,000 lb

15 ft

10 ft

50 ft

14. Determine the first moment of area of the shape below about both the x and y axes.

y

10

5

0

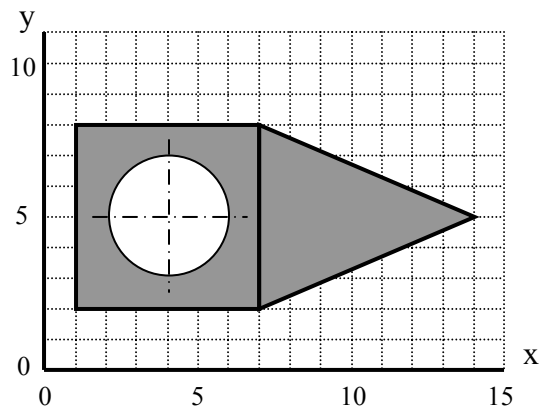0   5   10   15

x
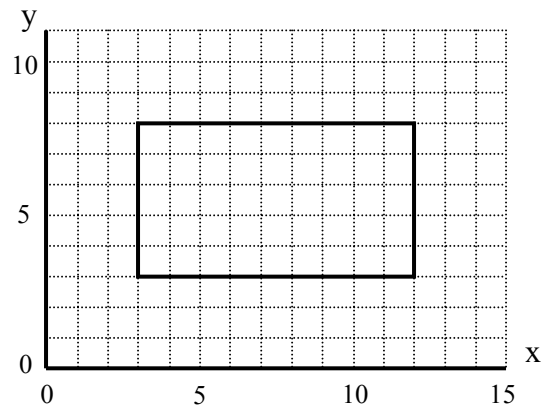
15. Determine the first moment of area of the shape below about both the x and y axes.

y

10

5

0

0   5   10   15

x
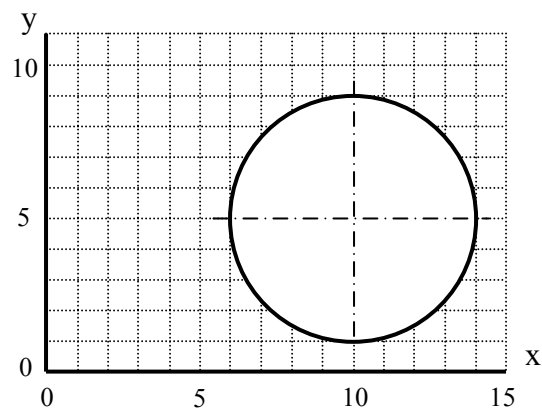
16.   Determine the centroid of the shape below about both the x and y axes.


17.   Determine the centroid of the shaded area about the x and y axes.

18.   The following diagram represents the distributed weight of a small ship.

a.   What is the difference between a distributed force and a resultant force?
b.   Calculate the resultant weight of the ship. The concept of weighted averages may be useful.
c.   At what point on the ship would the resultant weight be applied?

4 LT/ft

2.5 LT/ft

2 LT/ft   2 LT/ft

20 ft   30 ft   30 ft   40 ft

1 - 30
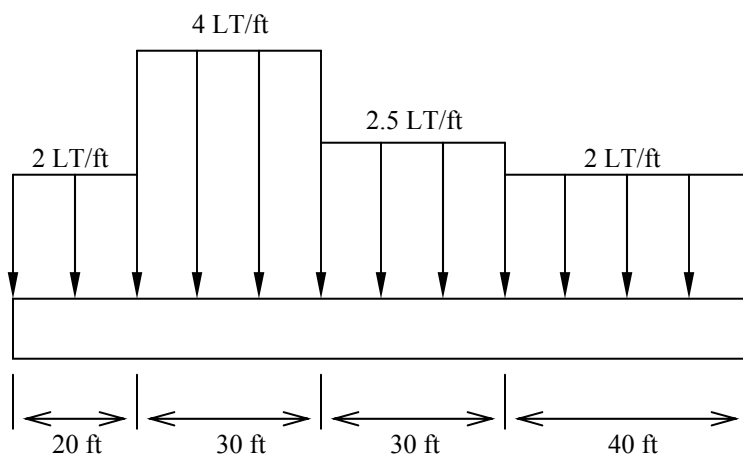
19.   Calculate the second moment of area of the below object about the following axes:

a.   Axis x_c-x_c (centroidal axis in the x-direction)

b.   Axis y_c-y_c (centroidal axis in the y-direction)

c.   Axis x-x (use of the parallel axis theorem is necessary)
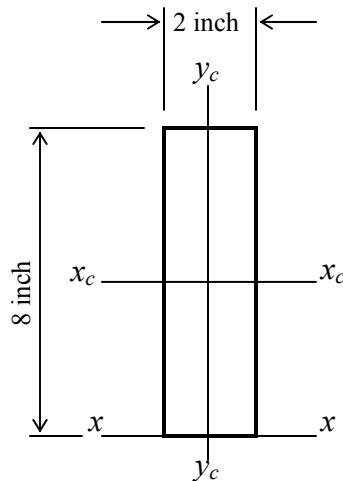
20.   Sketch a diagram showing the six degrees of freedom of a floating ship. Name each one on your diagram using the correct Naval Engineering terminology

21.   Describe the difference between a ship's list, heel, and roll.

22.   The diagram below shows the flow velocities past a submerged hydrofoil connected to an advanced marine vehicle. Use Bernoulli's Theorem to explain the forces being experienced by the hydrofoil.